Solving Proportions Through Multiple Representations

A **proportion** is an equation that states that two ratios are equivalent.

Numbers: $\dfrac{3}{5} = \dfrac{9}{15}$: The proportion is read, "3 is to 5 as 9 is to 15."

Algebra: $\dfrac{a}{b} = \dfrac{c}{d}$ where b and d are nonzero numbers.

Solving proportions through multiple representations will develop student understanding of proportions and develop relational and flexible thinking.

> Ask students, "What is a proportion?" This is a great opportunity for a think, pair, share. Accept reasonable responses. "A proportion is the equality of ratios."

Example 1: Solve the proportion $\dfrac{8}{3} = \dfrac{c}{18}$.

> Have students choral respond, "8 is to 3 as c is to 18."

$$\frac{8}{3} = \frac{c}{18}$$
$$\frac{8}{3} \cdot \frac{6}{6} = \frac{c}{18}$$
$$\frac{48}{18} = \frac{c}{18}$$
$$\therefore c = 48$$

To solve the proportion, look for a relationship. Because $3 \times 6 = 18$, multiply the fraction by the equivalent form of $1 = \dfrac{6}{6}$ to solve for *c*.

$$\frac{8}{3} = \frac{c}{18}$$
$$18 \cdot \frac{8}{3} = 18 \cdot \frac{c}{18}$$
$$\frac{2 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2}{3} = 18 \cdot \frac{c}{18}$$
$$2 \cdot 3 \cdot 2 \cdot 2 \cdot 2 = c$$
$$48 = c$$

Multiply both sides of the proportion by the LCM of the denominators to solve.

$$\frac{8}{3} = \frac{c}{18}$$
$$3 \bullet c = 8 \bullet 18$$
$$3c = 144$$
$$\frac{3c}{3} = \frac{144}{3}$$
$$c = \frac{3 \bullet 48}{3}$$
$$c = 48$$

Cross Products Property:
If $\dfrac{a}{b} = \dfrac{c}{d}$ where b and d are nonzero numbers, then $ad = bc$.

You Try #1: Solve the proportion $\dfrac{5}{2} = \dfrac{y}{10}$ three different ways, as in Example 1.

Example 2: Solve the proportion $\dfrac{4}{5} = \dfrac{x}{15}$.

Have students choral respond, "4 is to 5 as x is to 15."

$\dfrac{4}{5} = \dfrac{x}{15}$

$\dfrac{4}{5} = \dfrac{4+4+4}{5+5+5}$

$\therefore x = 4+4+4$

$x = 12$

Using the rules of equality and decomposition, "one 4 is to 5 as 3 groups of 4 are to 3 groups of 5."

$\dfrac{4}{5} = \dfrac{x}{15}$

4	8	12	16	20
5	10	15	20	25

Using extended ratios, we can see that "4 is to 5 as 12 is to 15."

$\therefore x = 12$

$\dfrac{4}{5} = \dfrac{x}{15}$

$\dfrac{4}{5} \cdot \dfrac{3}{3} = \dfrac{x}{15}$

$\dfrac{12}{15} = \dfrac{x}{15}$

$\therefore x = 12$

If two ratios are equal and their denominators are the same, their numerators must also be the same.

You Try #2: Solve the proportion $\dfrac{5}{7} = \dfrac{f}{28}$ three different ways, as in Example 2.

Example 3: Solve the proportion $\dfrac{33}{22} = \dfrac{6}{k}$.

Look for choral response opportunities when simplifying $\dfrac{33}{22}$.

$\dfrac{33}{22} = \dfrac{6}{k}$

$\dfrac{3 \cdot 11}{2 \cdot 11} = \dfrac{6}{k}$

$\dfrac{3}{2} = \dfrac{6}{k}$

$\dfrac{3}{2} \cdot \dfrac{2}{2} = \dfrac{6}{k}$

$\dfrac{6}{4} = \dfrac{6}{k}$

$\therefore k = 4$

By simplifying first, proportions can be solved much easier.

$\dfrac{33}{22} = \dfrac{6}{k}$

$\dfrac{3 \cdot 11}{2 \cdot 11} = \dfrac{6}{k}$

$\dfrac{3}{2} = \dfrac{6}{k}$

$\dfrac{2}{3} = \dfrac{k}{6}$
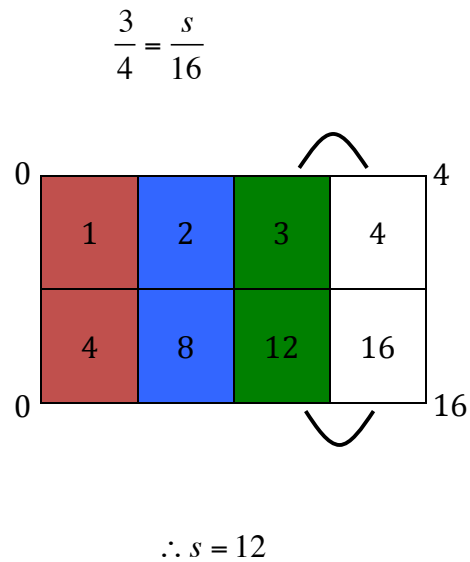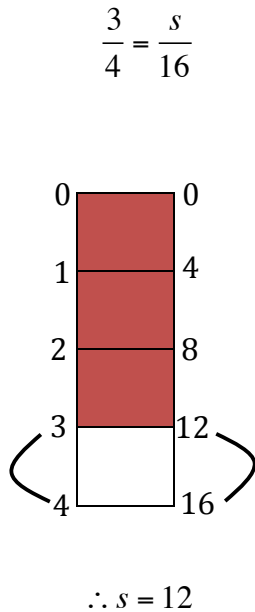
$6 \cdot \dfrac{2}{3} = 6 \cdot \dfrac{k}{6}$

$\dfrac{12}{3} = k$

$4 = k$

If two ratios are equal, then their reciprocals are equal.

You Try #3: Solve the proportion $\dfrac{28}{35} = \dfrac{12}{x}$ by simplifying first.

Example 4: Solve the proportion $\dfrac{3}{4} = \dfrac{s}{16}$ using bar models.

$$\frac{3}{4} = \frac{s}{16}$$

$$\frac{3}{4} = \frac{s}{16}$$

	1	2	3	4
	4	8	12	16

$\therefore s = 12$

$\therefore s = 12$

> Bar models are great visuals that show the relationships between parts and whole and the equality of each ratio.

You Try #4: Solve the proportion $\dfrac{u}{28} = \dfrac{4}{7}$ using a bar model.

Example 5: Set up a proportion to solve the Word Problem.

At a high school basketball game, the ratio of females to males in attendance is 2:3. If there are 120 female spectators in the gymnasium, what is the number of total spectators?

Set up a proportion to find the number of boys and then add the number of boys and girls together to determine the total number of spectators.

$$\frac{2}{3} = \frac{120}{x}$$

$$\frac{2}{3} \bullet \frac{60}{60} = \frac{120}{x}$$
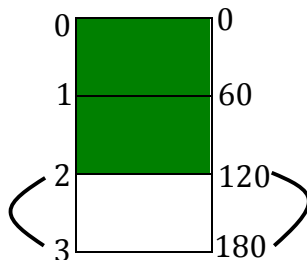
$$\frac{120}{180} = \frac{120}{x}$$

$$\therefore x = 180$$

So there are 180 boys.

$180 + 120$

$= 300$

$\therefore$ there are 300 total spectators

OR

$\therefore$ there are 180 boys

$180 + 120$

$= 300$

$\therefore$ there are 300 total spectators

You Try #5: Set up a proportion to solve the Word Problem.

A survey of sixth graders found that the ratio of students who prefer rock to hip-hop is 2 to 7. The total number of sixth graders surveyed was 72. How many of the students surveyed prefer rock?

Extra Problems: Solve each proportion three different ways.

1. $\dfrac{48}{30} = \dfrac{24}{x}$

2. $\dfrac{c}{9} = \dfrac{20}{45}$

3. $\dfrac{4}{7} = \dfrac{x}{42}$

4. $\dfrac{12}{16} = \dfrac{d}{40}$

5. $\dfrac{66}{x} = \dfrac{11}{15}$

Answers to Extra Problems:

1. $x = 15$

2. $c = 4$

3. $x = 24$

4. $d = 30$

5. $x = 90$

Explanation of Cross Products Property:

$$\frac{a}{b} = \frac{c}{d} \qquad b \text{ and } d \text{ must be nonzero numbers.}$$

$$bd\left(\frac{a}{b}\right) = bd\left(\frac{a}{d}\right)$$

$$\frac{abd}{b} = \frac{cbd}{d}$$

$$ad = cb$$

Worked-Out You Tries:

You Try #1: Solve the proportion $\dfrac{5}{2} = \dfrac{y}{10}$ three different ways.

$$\frac{5}{2} = \frac{y}{10}$$

$$\frac{5}{2} \cdot \frac{5}{5} = \frac{y}{10}$$

$$\frac{25}{10} = \frac{y}{10}$$

$$\therefore 25 = y$$

To solve the proportion, look for a relationship. Because $2 \times 5 = 10$, multiply the fraction by the equivalent form of $1 = \dfrac{5}{5}$ to solve for y.

$$\frac{5}{2} = \frac{y}{10}$$

$$10 \cdot \frac{5}{2} = 10 \cdot \frac{y}{10}$$

$$\frac{2 \cdot 5 \cdot 5}{2} = \frac{10y}{10}$$

$$5 \cdot 5 = y$$

$$25 = y$$

Multiply both sides of the proportion by the LCM of the denominators to solve.

$$\frac{5}{2} = \frac{y}{10}$$

$$2 \cdot y = 5 \cdot 10$$

$$2y = 50$$

$$\frac{2y}{2} = \frac{50}{2}$$

$$y = 25$$

Cross Product Property:

If $\dfrac{a}{b} = \dfrac{c}{d}$ where b and d are nonzero numbers, then $ad = bc$.

You Try#2: Solve the proportion $\dfrac{5}{7} = \dfrac{f}{28}$ three different ways.

$$\frac{5}{7} = \frac{f}{28}$$

$$\frac{5}{7} = \frac{5+5+5+5}{7+7+7+7}$$

$$\therefore f = 5+5+5+5$$
$$f = 20$$

Using the rules of equality and decomposition, one 5 is to 7 as 4 groups of 5 are to 4 groups of 7.

$$\frac{5}{7} = \frac{f}{28}$$

5	10	15	20	25
7	14	21	28	35

Using extended ratios, we can see that 5 is to 7 as 20 is to 28.

$$\frac{5}{7} = \frac{f}{28}$$

$$\frac{5}{7} \cdot \frac{4}{4} = \frac{f}{28}$$

$$\frac{20}{28} = \frac{f}{28}$$

$$\therefore f = 20$$

If two ratios are equal and their denominators are the same, their numerators must also be the same.

You Try #3: Solve the proportion $\dfrac{28}{35} = \dfrac{12}{x}$ by simplifying first.

$$\frac{28}{35} = \frac{12}{x}$$

$$\frac{4 \cdot \cancel{7}}{5 \cdot \cancel{7}} = \frac{12}{x}$$

$$\frac{4}{5} = \frac{12}{x}$$

$$\frac{4}{5} \cdot \frac{3}{3} = \frac{12}{x}$$

$$\frac{12}{15} = \frac{12}{x}$$

$$\therefore x = 15$$

By simplifying first, proportions can be solved much easier.

$$\frac{28}{35} = \frac{12}{x}$$

$$\frac{4 \cdot \cancel{7}}{5 \cdot \cancel{7}} = \frac{12}{x}$$

$$\frac{4}{5} = \frac{12}{x}$$

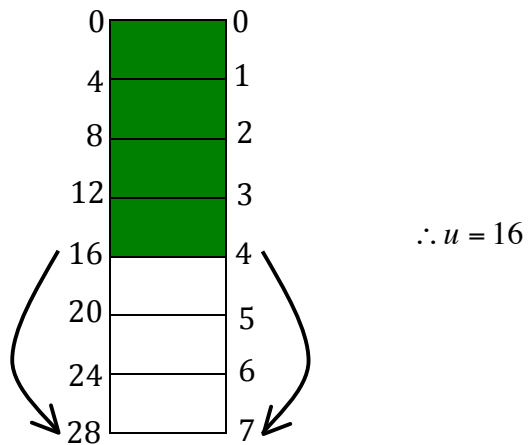$$\frac{5}{4} = \frac{x}{12}$$

$$12 \cdot \frac{5}{4} = 12 \cdot \frac{x}{12}$$

$$\frac{60}{4} = \frac{12x}{12}$$

$$15 = x$$

If two ratios are equal, then their reciprocals are equal.

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You Try #4: Solve the proportion $\dfrac{u}{28} = \dfrac{4}{7}$ using a bar model.

$\therefore u = 16$

You Try #5: Set up a proportion to solve the Word Problem.

A survey of sixth graders found that the ratio of students who prefer rock to hip-hop is 2 to 7. The total number of sixth graders surveyed was 72. How many of the students surveyed prefer rock.

$$\frac{2}{9} = \frac{x}{72}$$
$$\frac{2}{9} \cdot \frac{8}{8} = \frac{x}{72}$$
$$\frac{16}{72} = \frac{x}{72}$$
$$x = 16$$

> The ratio of students who prefer rock to hip-hop is 2 to 7 which means that 2 out of every nine prefer rock. That's the ratio that must be used in the proportion.

$\therefore 16$ students surveyed prefer rock.

Answers to Warm-Up Problems:

CST/CAHSEE: D

Current: $f = 24$ Review: $x = 32$ Other: $\dfrac{2}{3}$